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A TWO-DIMENSIONAL OMEGA EQUATION FOR THE

1000-700 MB LAYER WITH DIABATIC HEATING

by

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ABSTRACT

A two-dimensional omega equation is derived by combination of the vorticity and thermodynamic equations. The desired omega is then taken to be the logarithmic average in the 1000-700 mb layer. A diabatic term, after Laevastu, for oceanic areas only is included to deduce the empirical temperature and vapor-pressure changes associated with sensible and latent heating in the maritime layers. Over both continental and oceanic areas a frictional vorticity sink is included in order that excessive energy cannot be generated over the ocean. Among other novel features is the use of the Holl static-stability parameter which affords vertical consistency with analyses prepared by Fleet Numerical Weather Facility.

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LIST OF SYMBOLS

Scalar Quantities

T	temperature
T_A	surface air temperature
T_D	surface dew-point temperature
T_W	sea water temperature (surface)
T_{WB}	wet-bulb temperature
T_C	condensation level temperature
θ	potential temperature
e_A	surface vapor pressure
e_S	surface saturation vapor pressure
e_W	vapor pressure over water
e_C	condensation level vapor pressure
P	pressure
P_A	surface pressure
P_C	condensation level pressure
Z	height above mean sea level
Z_T	terrain height
D	Z (actual)- Z (standard atmosphere)
η	absolute vorticity
ζ	relative vorticity
f	coriolis force
ω	vertical motion of a pressure surface
ω_{LO}	vertical motion at the lower boundary
ω_F	vertical motion due to frictional effects

ω_T	vertical motion due to terrain effects
σ	stability parameter
σ_H	Holl stability parameter
V	wind speed
V_{10}	10-meter wind speed
m	mass
ρ	density
t	time
C_D	coefficient of drag
$\dot{Q}$	heating rate per unit mass
Q_c	heat flux for the inversion case
α	cross isobar angle
γ_m	moist adiabatic lapse rate

Constants

$\gamma_d = g/C_p$	dry adiabatic lapse rate
γ_D	dew-point lapse rate
R	gas constant for dry air
C_p	specific heat at constant pressure
L	latent heat of vaporization
g	acceleration of gravity

Vector-scaler operators

$\mathbb{V}$	velocity
$\mathbb{V}_{go}$	geostrophic wind velocity
∇A	gradient of A
$\nabla^2 A$	Laplacian of A
$\mathbb{J}(A,B)$	Jacobian of A and B

1. Introduction

Little practical use has been made of vertical motions, $\omega = dp/dt$, in short-range forecasting due to the complicated and lengthy computations required. Vertical motion computations are usually the by-product of multilevel baroclinic models and single values must be extracted only after going through the entire three-dimensional procedure. In this paper a two-dimensional vertical motion equation is developed using vertically integrated parameters with an assumed vertical motion profile. The layer-mean omega is especially useful for short-range thickness forecasts and the 1000-700 mb layer has been selected for study. Derivation of the ω-equation is similar to that by Thompson [18] except a diabatic heating term providing a mechanism for development over oceanic areas has been included. Due to the empiricisms employed the non-elliptic conditions mentioned by Pedersen [16] do not arise.

2. The Basic Omega Equation

The diagnostic omega equation is derived by standard means from a vorticity equation and a thermodynamic equation which retains the diabatic term. Equation (1) is the time derivative of the first law of thermodynamics in (x, y, p, t) coordinates.

$$\frac{\partial T}{\partial t} + \mathbb{V}\cdot\nabla T + \frac{T}{\theta}\frac{\partial\theta}{\partial p}\,\omega = \frac{\dot{Q}}{C_p} \qquad (1)$$

Substituting, $T = -\frac{g\,\partial z}{R\,\partial \ln p}$, which comes from the hydrostatic assumption and the ideal gas law; then dividing by, -g/R, yields equation (2).

$$\frac{\partial}{\partial t}\left(\frac{\partial z}{\partial \ln p}\right) + \mathbb{V}\cdot\nabla\left(\frac{\partial z}{\partial \ln p}\right) - \frac{RT\,\partial\theta}{g\theta\,\partial p}\,\omega = -\frac{R\dot{Q}}{gC_p} \qquad (2)$$

Operating on equation (2) with the Laplacian gives the final form of the thermodynamic equation as shown by equation (3).

$$\frac{\partial}{\partial \ln p}\nabla^2\left(\frac{\partial z}{\partial t}\right) + \nabla^2\left(\mathbb{V}\cdot\nabla\left(\frac{\partial z}{\partial \ln p}\right)\right) + \frac{1}{pg}\nabla^2\sigma_H\omega = -\frac{R}{gC_p}\nabla^2\dot{Q} \qquad (3)$$

Here, $\sigma_H = -Rp\frac{T\,\partial\theta}{\theta\,\partial p}$, is the Holl stability parameter to be further discussed in section (4).

The vorticity equation in pressure coordinates is shown by equation (4).

$$\frac{\partial \zeta}{\partial t} + \mathbb{V}\cdot\nabla(\zeta+f) + \omega\frac{\partial}{\partial p}(\zeta+f) =$$
$$(\zeta+f)\frac{\partial\omega}{\partial p} + \left(\frac{\partial\omega\,\partial u}{\partial y\,\partial p} - \frac{\partial\omega\,\partial v}{\partial x\,\partial p}\right) \qquad (4)$$

As presented by Thompson [18] and numerous other writers the last term of the left side of (4) is approximately equal to that on the right side, and the two terms are henceforth deleted. Then, a useful form of the vorticity equation is arrived at (equation 5) by making the geostrophic assumption for vorticity and for velocity, $\zeta_g = \frac{g}{f}\nabla^2 z$, $\mathbb{V}_g = \frac{g}{f}\nabla z$, and by taking the logarithmic pressure derivative.

$$\frac{\partial}{\partial \ln p}\nabla^2\left(\frac{\partial z}{\partial t}\right) + \frac{\partial}{\partial \ln p}\mathbb{J}(z,\eta) - \frac{f\eta}{g}\frac{\partial}{\partial \ln p}\left(\frac{\partial \omega}{\partial p}\right) = 0 \qquad (5)$$

Here, $\eta = \zeta_g + f$, is the absolute vorticity. Next subtract (5) from (3), and the result is the omega equation:

$$\nabla^2(\sigma_H \omega) + f\eta p^2 \frac{\partial^2 \omega}{\partial p^2} = pg\frac{\partial}{\partial \ln p}\mathbb{J}(z,\eta) - pg\nabla^2\left(\mathbb{V}\cdot\nabla\left(\frac{\partial z}{\partial \ln p}\right)\right) - p\frac{R}{c_p}\nabla^2\dot{Q} \qquad (6)$$

3. Vertical Distribution of Pressure and Omega

Instead of the distribution of pressure indexes normally considered, that employed here is given by,

$$\frac{P_n}{P_0} = \frac{P_0}{P_0}\left(2^{-\frac{n}{4}}\right) = 2^{-\frac{n}{4}}$$

where n is an arbitrary pressure level. The 1000-500 mb layer is divided into sub-layers as shown by Figure (1). The resultant pressures, while essentially logarithmic, bear values nearly identical to those of the mandatory levels.

4 ---------	$P_4 = P_0 2^{-1}$	=	500.0 mb
3 ---------	$P_3 = P_0 2^{-\frac{3}{4}}$	=	594.6 mb
2 ---------	$P_2 = P_0 2^{-\frac{1}{2}}$	=	707.1 mb
1 ---------	$P_1 = P_0 2^{-\frac{1}{4}}$	=	840.9 mb
0 ---------	$P_0 = P_0 2^{0}$	=	1000.0 mb

Figure 1. Pressure Distribution in the Vertical

The values of $\omega = dP/dt$, the "vertical" velocity in pressure coordinates, is assumed to vary parabolically in the vertical with a profile defined by equation (7), which extends nearly to the 500 mb level.

$$\omega = A\left(\ln\frac{P}{P_0}\right)^2 + B\left(\ln\frac{P}{P_0}\right) \tag{7}$$

For boundary conditions it is assumed that $\omega = 0$ at $p = p_0$ (the kinematic boundary condition at the assumed surface of the earth, p_0). Also for convenience it is assumed that at p_4 $\partial\omega/\partial p = 0$, i.e. that the 500-mb divergence is zero. Then, upon differentiation of equation (7) with the 500-mb divergence taken to be zero, equation (8) results. It should be noted that if the value $(\partial\omega/\partial p) = 0$ is assumed zero at $p = p_3$ the value of B is altered by only 5%.

$$\left(\frac{\partial\omega}{\partial p}\right)_4 = \left[2A\left(\ln\frac{P}{P_0}\right)\frac{\partial}{\partial P}\left(\ln\frac{P}{P_0}\right) + B\frac{\partial}{\partial P}\left(\ln\frac{P}{P_0}\right)\right]_{P=P_4} = 0$$

or, $$2A\left(\ln\frac{P_4}{P_0}\right)\left(\frac{1}{P_4}\right) + B\left(\frac{1}{P_4}\right) = 2A\left(\ln 2^{-1}\right)\left(\frac{1}{P_4}\right) + B\left(\frac{1}{P_4}\right) = 0 \tag{8}$$

Therefore, B=1.3863A, and the omega profile in terms of A is given by equation (9).

$$\omega = A\left[\left(\ln\frac{P}{P_0}\right) + 1.3863\left(\ln\frac{P}{P_0}\right)\right] \tag{9}$$

Since, $\omega_0 = 0$, the ratio between ω_1, and ω_2, defines the omega profile (equation 10) for the 0-2 layer (see Figure 2).

$$\frac{\omega_2}{\omega_1} = \frac{-\frac{1}{2}A\ln 2\left(-\frac{1}{2}\ln 2+1.3863\right)}{-\frac{1}{4}A\ln 2\left(-\frac{1}{4}\ln 2+1.3863\right)} = 1.7143 \qquad (10)$$

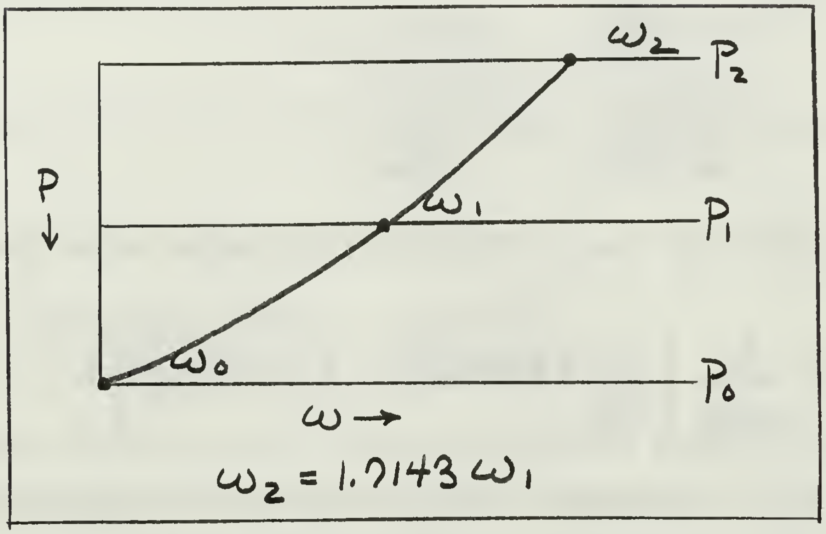

Figure 2. The Omega Profile

For the pressure-range considered (see Figure 2), the layer-logarithmic-mean, $\bar{\omega} = 0.950\omega_1$, so that ω_1, will be used for $\bar{\omega}$. Note finally, the profile assumed here applies only to the large-scale adiabatic, frictionless component of the vertical velocity (that is terrain irregularities are considered absent).

4. Vertical Integration of the Omega Equation

The Holl stability parameter [11] is used since it is one of the vertical-consistency requirements used by FNWF (Fleet Numerical Weather Facility) and this study uses FNWF processed data. The Holl parameter is a modification of the standard stability parameter used here, as shown by equations (11), (12), and (13).

$$\sigma_H = Rp\sigma = -Rp\frac{T\partial\theta}{\theta\partial p} = Rp\left(\frac{RT}{c_p p} - \frac{\partial T}{\partial p}\right) \tag{11}$$

Letting, $RT = -g\frac{\partial z}{\partial \ln p}$ gives,

$$\sigma_H = -\frac{Rg\,\partial z}{C_p\,\partial \ln p} + R\frac{\partial T}{\partial \ln p} \tag{12}$$

Equation (12) may now be finite differenced over the 0-2 layer.

$$\overline{\sigma}_H = \frac{R}{\ln\frac{P_0}{P_2}}\left[\frac{R}{C_p}\left(z_2 - z_0\right) - \left(T_0 - T_2\right)\right] \tag{13}$$

Equation (13) represents the Holl stability parameter in finite-difference form. In effect, $\overline{\sigma}_H$, by (13) gives the logarithmic-pressure "average" over the 0-2 layer. This use of $\overline{\sigma}_H$ is compatible with the assumed logarithmic pressure distribution and will be used throughout the analysis which follows.

The omega equation (6) will now be integrated term by term over the isobaric layer (Po, P_2). From (6), one may rewrite the ω-equation as:

$$\underset{(A)}{\nabla^2(\sigma_H\omega)} + \underset{(B)}{f\eta p^2\frac{\partial^2\omega}{\partial p^2}} = \underset{(C)}{pg\frac{\partial}{\partial \ln p}J(z,\eta)}$$

$$-\underset{(D)}{pg\nabla^2\left(\mathbb{V}\cdot\nabla\left(\frac{\partial z}{\partial \ln p}\right)\right)} - \underset{(E)}{p\frac{R}{C_p}\nabla^2\dot{Q}}$$

whose parts (A),...., (E) will now be discussed individually.

(A) The first term is approximated by substituting layer mean values: $\nabla^2(\bar{\sigma}_H \omega_1)$, $\omega_1 \doteq \bar{\omega}$

(B) With the mathematical relation, $p^2 \frac{\partial^2 \omega}{\partial p^2} = \frac{\partial^2 \omega}{\partial (\ln p)^2} - \frac{\partial \omega}{\partial \ln p}$, the second term becomes,

$$f\eta_1\left(\frac{\partial^2 \omega}{\partial (\ln p)^2} - \frac{\partial \omega}{\partial \ln p}\right)$$

whose center-finite-differenced form over the 0-2 layer becomes:

$$f\eta_1\left(\frac{\omega_2 - 2\omega_1 + \omega_0}{(\frac{1}{2}\ln P_0/P_2)^2} + \frac{\omega_2 - \omega_0}{\ln P_0/P_2}\right) \qquad (14)$$

Then, using the specified vertical motion profile the relation $\omega_2 = 1.7143\omega_1$, is substituted. However, it is desired to introduce lower boundary effects due to friction and terrain by letting $\omega_0 = \omega_{L0}$, the ω_{L0} term then being predetermined and placed on the forcing function side of the ω-equation. Expression (15) below is the final form of (14) including effects of the lower boundary.

$$\text{Term (B)} \doteq f\eta_1\left(-\frac{0.2857\omega_1}{(\frac{1}{2}\ln P_0/P_2)^2} + \frac{1.7143\omega_1}{\ln P_0/P_2}\right) + f\eta_1\left(\frac{\omega_{L0}}{(\frac{1}{2}\ln P_0/P_2)^2} - \frac{\omega_{L0}}{\ln P_0/P_2}\right) \qquad (15)$$

The lower boundary, ω_{L0}, will be discussed in section (5).

(C) Term (C) in the geostrophic-diagnostic model represents geostrophic advection of absolute vorticity. Averaging in the vertical with respect to the logarithm of pressure leads to equation (16):

$$\frac{1}{\ln\frac{p_0}{p_2}}\int_{p_2}^{p_0} pg\frac{\partial \mathbb{J}}{\partial \ln p}(z,\eta)\Delta\ln p \doteq \frac{p_1 g}{\ln\frac{p_0}{p_2}}\int_{p_2}^{p_0}\frac{\partial}{\partial \ln p}\mathbb{J}(z,\eta)\Delta\ln p \qquad (16)$$

The derivation of equation (16) makes use of the mean value theorem, so that $p \doteq p_1$ is represented by p_1 and may be removed from within the integral. Equation (16) then becomes,

$$\frac{p_1 g}{\ln p_0/p_2}\mathbb{J}(z,\eta)\Bigg|_{p_2}^{p_0} = \frac{p_1 g}{\ln p_0/p_2}\mathbb{J}(z_0,\eta_0) - \frac{p_1 g}{\ln p_0/p_2}\mathbb{J}(z_2,\eta_2) \qquad (17)$$

(D) Term (D) involves the geostrophic advection of thickness within the logarthmic pressure integral. Its value is well approximated by equation (18):

$$\frac{1}{\ln p_0/p_2}\int_{p_2}^{p_0} pg\nabla^2\left(\mathbb{V}\cdot\nabla\left(\frac{\partial z}{\partial \ln p}\right)\right)\Delta\ln p \doteq \frac{p_1 g^2}{f\ln p_0/p_2}\nabla^2\int_{p_2}^{p_0}\mathbb{J}\left(z,\frac{\partial z}{\partial \ln p}\right)\Delta\ln p \qquad (18)$$

the right side of which gives equation (19).

$$\frac{p_1 g^2}{f\ln p_0/p_2}\nabla^2\mathbb{J}(z_1,\Delta z) = \frac{p_1 g^2}{f\ln p_0/p_2}\nabla^2\mathbb{J}(z_1, z_2 - z_0) \qquad (19)$$

Equation (19) shows that the geostrophic advecting wind may be taken arbitrarily as the level (1) wind.

(E) The diabatic heating term will be vertically integrated in section (6).

The integrated form of the resulting omega equation, subject to the lower and upper boundary conditions (at p_0 and p_4) becomes,

$$\nabla^2(\bar{\sigma}_H\omega_1) + \frac{f\eta_1}{\bar{\sigma}_H}\left(\frac{1.7143\omega_1}{\ln P_0/P_2} - \frac{0.2857\omega_1}{(\frac{1}{2}\ln P_0/P_2)^2}\right)\bar{\sigma}_H =$$

$$-f\eta_1\left(\frac{\omega_{L0}}{(\frac{1}{2}\ln P_0/P_2)^2} - \frac{\omega_{L0}}{\ln P_0/P_2}\right) + \frac{P_1 g}{\ln P_0/P_2}\left(J(z_0,\eta_0) - J(z_2,\eta_2)\right)$$

$$+\frac{P_1 g^2}{f\ln P_0/P_2}\nabla^2 J(z_1, z_2 - z_0) - p\frac{R}{C_p}\nabla^2\dot{Q} \tag{20}$$

Equation (20) is a Helmholtz-type equation in the variable, $\bar{\sigma}_H\omega_1$. This grouping, $\bar{\sigma}_H\omega_1$, precludes making the usual simplifying approximation of most three-dimensional models. (See for example, Haltiner et al [9]).

$$\nabla^2(\sigma\omega) = \sigma\nabla^2\omega + \omega\nabla^2\sigma + 2\nabla\sigma\cdot\nabla\omega \doteq \sigma\nabla^2\omega$$

The final solution of equation (20) is to be divided by $\bar{\sigma}_H$ leaving $\omega_1 \doteq \bar{\omega}$, where $\bar{\omega}$ is the mean vertical motion which approximates the 850 mb vertical motion.

5. The Lower Boundary Condition

Vertical motion at the lower boundary ω_{L0}, is the sum of a terrain and a friction term, $\omega_{L0} = \omega_T + \omega_F$.

Here, ω_T, the terrain-effected vertical motion may be described by equation (21) from Berkofski and Bertoni [2].

$$\omega_T = -\rho g \mathbb{V}_T \cdot \nabla z_T \tag{21}$$

Terrain height, z_T, is a smoothed field used operationally by FNWF. The wind velocity at terrain level, $\mathbb{V}_T$, will be arrived at by an objective procedure to be described below.

The friction term ω_F, as shown by equation (22) is a simplified version of Cressman's formula used by Haltiner et al [9].

$$\omega_F = -\rho_T \frac{g}{f} C_D V_T \zeta_T \tag{22}$$

C_D is the geostrophic drag coefficient derived by Cressman [7], the field of which is in regular use at FNWF. The field of C_D is a set of constants, one for each (ij) grid-point in the Northern Hemisphere.

Since the T subscripts infer application at terrain height, values are used that approximate the particular terrain heights involved. This is done by dividing the 0-2 layer into three terrain-height dependent cases.

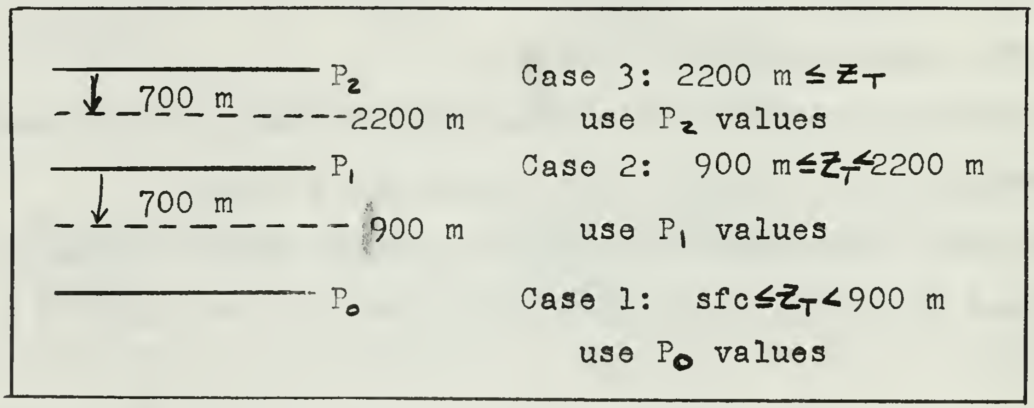

Figure 3. Three Cases of the Lower Boundary

Division of the 0-2 layer as shown by figure (3) is dependent upon the gradient level above terrain, since $\mathbb{V}_T$ is approximated by some geostrophic wind throughout. As a basis for the selection of the particular geostrophic wind, $\mathbb{V}_g$, note that the gradient level in a neutral atmosphere occurs at

approximately 700 m above terrain level for a wide range of surface roughness, as shown by Blackadar [3]. Therefore, vertical velocity at the lower boundary is case (1), if the terrain height is 900 m or lower. This values corresponds to a "standard atmosphere" height which is 700m or more below the (1) level, and (0) level parameters are then used to compute the lower boundary. Case (2) is to be used if the terrain height Z_T lies within a gradient-level range of p, but not p_2. Case (3) occurs for all terrain heights within 700 m of p_2.

Equation (23) is the terrain-induced vertical velocity at the lower boundary after application of the geostrophic assumption.

$$(\omega_{LO})_n = -\rho_n \frac{g^2}{f} J(D_n, Z_T) - \rho_n \frac{g^3}{f^3} C_D \left| \mathbb{K} \times D_n \right| \nabla^2 D_n \quad (23)$$

Here, the n subscript refers to the levels 0, 1, and 2. Densities are standard atmosphere values for the respective levels.

6. The Diabatic Heating Term

Diabatic heating in this model results from an exchange of heat between the sea surface and the atmosphere. The exchange is divided into three cases: neutral, convective, and inversion, the division being dependent upon the sea-air temperature difference, $T_w - T_a$.

The neutral case ($0^\circ C \leq T_W - T_A < 3^\circ C$) results in no net transport of heat across the air-ocean interface and the

diabatic heating term is arbitrarily taken as zero when this condition prevails.

The convective case ($T_W - T_A \geq 3^{\circ}C$) results in a net heating of the atmosphere. The convective atmosphere is assumed to consist of a dry adiabatic lapse rate to the condensation level and a moist adiabatic lapse rate above. This particular model of the convective case is discussed by Burke [5] in his paper on the transformation of cP to mP air. Heat gain by the atmosphere is manifested by a layer-mean temperature gain induced by a one-hour trajectory of surface air over a warmer sea-surface, and the layer temperature change is shown by figure (4).

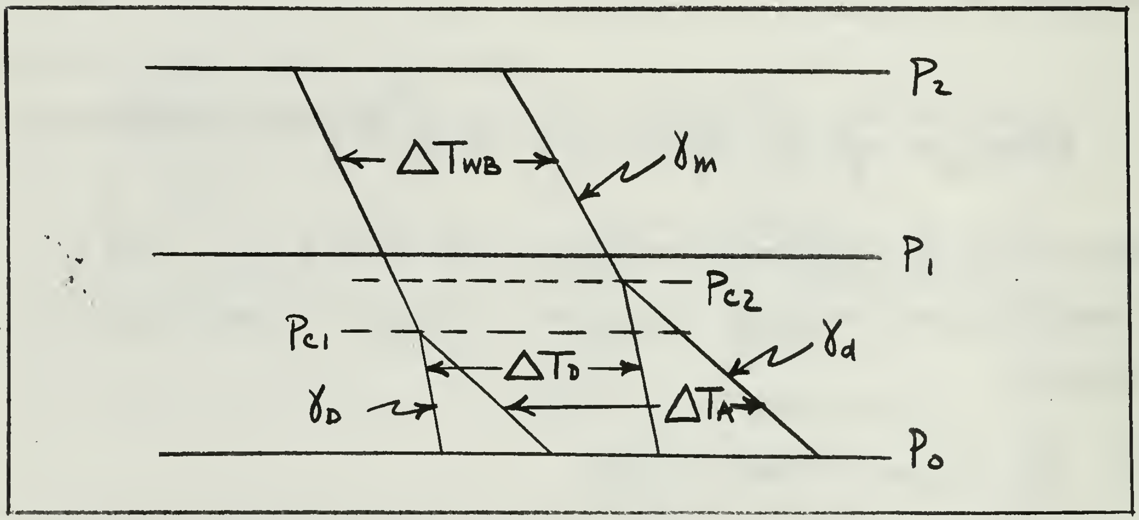

Figure 4. Model of the Convective-Atmosphere Modification

Heating is composed of: (1) sensible heat from the surface to the condensation level and (2) latent heating above this level. This result follows by an analysis of the Burke model and its implications reguarding heat transport. Martin [14] has shown that the large-scale transports of heat have

already been included in those terms of equation (6) in which $\dot{Q}$ does not specifically appear. Thus $\dot{Q}/C_p$, for use in equation (6) is given by the local temperature change.

$$\dot{Q} = C_p \frac{\Delta T}{\Delta t} = \left(C_p \frac{\Delta T_A}{\Delta t}\right)_{SENSIBLE} + \left(C_p \frac{\Delta T_{WB}}{\Delta t}\right)_{LATENT} \qquad (24)$$

Equation (25) is the diabatic heating term (E) referred to in equation (6).

$$-Rp\nabla^2 \frac{\dot{Q}}{C_p} = -Rp\nabla^2\left[\left(\frac{\Delta T_A}{\Delta t}\right) - \left(\frac{\Delta T_{WB}}{\Delta t}\right)\right] \qquad (25)$$

Integrating logarithmically in the vertical yields the final form of the diabatic term as shown by equation (26).

$$-\frac{R}{\ln P_c/P_2}\nabla^2\int_{P_2}^{P_c} p \frac{\Delta T_{WB}}{\Delta t}\,\delta \ln p - \frac{R}{\ln P_A/P_c}\nabla^2 \int_{P_c}^{P_A} p\frac{\Delta T_A}{\Delta t}\,\delta\ln p$$

$$= -\frac{R}{\ln P_A/P_2}\nabla^2\left[(P_c-P_2)\frac{\Delta T_{WB}}{\Delta t} + (P_A-P_c)\frac{\Delta T_A}{\Delta t}\right] \qquad (26)$$

In each of equations 24, 25, and 26, the operator $\frac{\Delta}{\Delta t}$ represents the finite-difference version of the local time derivitive, while, P_c , represents a convective-condensation level pressure.

According to the model depicted in figure (4), the modification is largely determined by the static stability which in turn largely depends on $\overline{\sigma_H}$. As an assumed infinite source of heat, the ocean will first modify a thin surface layer of air and then, due to convective activity, will distribute the modification (temperature change) throughout the 0-2 layer. An overestimation of heat exchange should be expected with

this particular process since convection is not instantaneous This overestimation may be partially balanced by heat lost from the top of the 0-2 layer. However, the non-inclusion of a moisture-continuity equation may be more serious in regions of upper ridges.

The inversion case ($T_W - T_A < 0$) or cooling case, where heat is lost from the atmosphere to the ocean is defined by an empiricism from Laevastu [12].

$$Q_c = 3.0\, V_{10} \left(T_W - T_A \right) \text{ gm-cal/cm}^2\text{-24hrs} \tag{27}$$

Equation (27) represents the heat flux across the 10-meter level. This is an empiricism requiring the 10-meter wind, V_{10}, which will be approximated by making a frictional correction to the 1000 mb geostrophic wind as shown by equation (28):

$$V_{10} = 0.5 \frac{g}{f} \left| \mathbb{K} \times \nabla D_0 \right| \tag{28}$$

The factor 0.5 is due to surface frictional effects and will be discussed in section (7).

Before integrating equation (27) over the 0-2 layer the initial distribution within the layer must be determined. Normally cooling effects due to inversion conditions are confined to a surface layer only several hundred meters thick, however, since the layer mean cooling effect is desired over the entire 0-2 layer it will be assumed that cooling is distributed throughout the 0-2 layer. Thus, dividing equation

(27) by the mass of a column of air extending from the surface pressure P_A to P_2, with total mass, $m = (P_A - P_2)/g$, gives the mean temperature change in a vertical column extending from P_A to P_2 as,

$$\frac{\dot{Q}}{C_p} = \frac{Q_c\, g}{C_p(P_A - P_2)} \tag{29}$$

Finally we have,

$$-R_p \nabla^2 \frac{\dot{Q}}{C_p} = -\frac{R g \nabla^2}{C_p \ln\frac{P_A}{P_2}} \int_{P_2}^{P_A} P \frac{Q_c\, \Delta P}{(P_A - P_2) P} = -\frac{R g}{C_p \ln \frac{P_A}{P_2}} \nabla^2 Q_c \tag{30}$$

which is the final result for the inversion case assuming neglible heat release from fog-droplet condensation.

7. State-Change Parameters for the Convective Case

Air-temperature change and wet-bulb temperature change equations must be derived for use in equation (26). The state-change equations (31) and (32) were initially developed by Mosby [15], Åmot [1], and more recently by Boyum [4]. However, in a FNWF study by Carstensen and Laevastu [6] the following best-fit equations were found to give hourly changes with a high degree of skill:

$$\frac{\Delta T_A}{\Delta t} = 0.12(T_w - T_A) - 0.10 - \mathbb{V}_{10} \cdot \nabla T_A \quad {}^\circ C/hr \tag{31}$$

$$\frac{\Delta e_A}{\Delta t} = 0.15(e_w - e_A) - 0.18 - \mathbb{V}_{10} \cdot \nabla e_A \quad mb/hr \tag{32}$$

Equation (31) is substituted directly into the sensible-heat term of equation (26) while the wet-bulb temperature change contributes directly to the latent-heat term. In

computing $\frac{\Delta T_{WB}}{\Delta t}$ it is necessary to include the small increment of temperature found by proceeding along a mixing ratio isopleth from P_{C_1} to P_{C_2} then down a moist adiabat to P_{C_1} (see figure 5). Equation (33) shows this relationship.

$$\Delta T_{WB} = \Delta T_D - (\gamma_D - \gamma_m)\Delta Z_{pc}\ , \quad \gamma_D = 1.600 \frac{°C}{Km} \tag{33}$$

Here ΔZ_{pc} is the one-hour height change of the condensation heights, determined by the height of intersection of the dry adiabat and the mixing ratio isopleth based upon the temperature - dew-point spread at the surface.

$$\Delta Z_{pc} = Z_{pc2} - Z_{pc1} = \left(\frac{T_{A2} - T_{D2}}{\gamma_d - \gamma_D}\right) - \left(\frac{T_{A1} - T_{D1}}{\gamma_d - \gamma_D}\right) \tag{34}$$

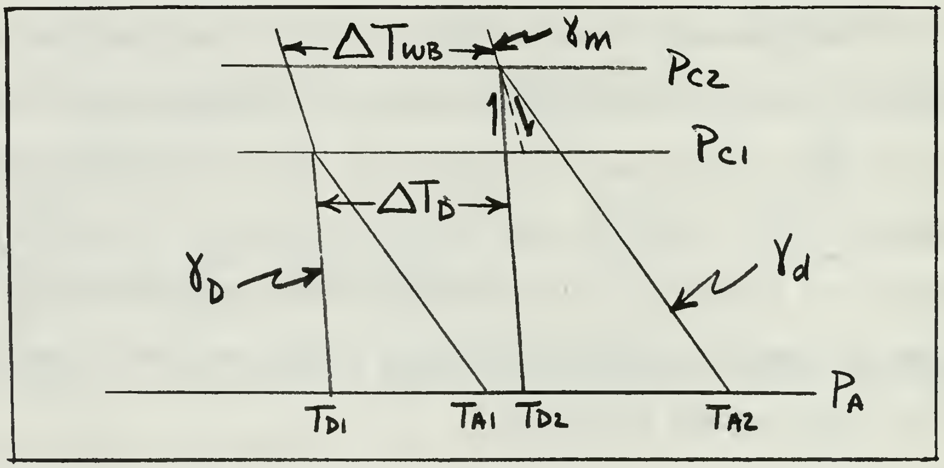

Figure 5. ΔT_{WB} as a Function of ΔT_D

Substituting equation (34) into (33) gives the local rate of change of the wet-bulb temperature as a function of <u>dew-point</u> and <u>air-temperature change</u>,

$$\frac{\Delta T_{WB}}{\Delta t} = \frac{\Delta T_D}{\Delta t} + \left(\frac{\gamma_D - \gamma_m}{\gamma_D - \gamma_d}\right)\left(\frac{\Delta T_A}{\Delta t} - \frac{\Delta T_D}{\Delta t}\right) \tag{35}$$

The dew-point temperature change ΔT_D is found by taking the time differential of equation (36) below [10],

$$T_A - T_D \doteq \frac{0.622L}{C_p P}(e_S - e_A) \tag{36}$$

Here, e_s is the saturated vapor pressure and e_A the observed value, both at 10-meters. The finite-difference form of the time differential of dew-point resulting from equation (36) then follows:

$$\frac{\Delta T_D}{\Delta t} \doteq \frac{\Delta T_A}{\Delta t} + \frac{0.622L}{C_p P}\frac{\Delta e_A}{\Delta t} - \frac{0.622L}{C_p P}\frac{\Delta e_s}{\Delta t} \tag{37}$$

The vapor pressure change, $\frac{\Delta e_A}{\Delta t}$, is given by equation (32). The time rate of change of saturated vapor pressure, $\frac{\Delta e_s}{\Delta t}$, is calculated using the Clausius - Clapeyron equation:

$$\frac{\Delta e_s}{\Delta t} = \frac{0.622 L e_s}{R T_A^2}\frac{\Delta T_A}{\Delta t} \tag{38}$$

$$e_s = 6.107 \exp\left[\frac{5418.0}{273.16} - \frac{5418.0}{T_A}\right] \tag{39}$$

Equation (39) is an integrated form of the Clausius - Clapyron equation.

There are still some unsolved parameters in the system. The moist adiabatic lapse rate, γ_m, cannot be considered a constant with respect to pressure even in the limited operational range of this study. At the condensation level, P_c, the moist adiabat is calculated from the following [10]:

$$\gamma_m = \gamma_d \left(\frac{P_c + \frac{0.622 L e_c}{R T_c}}{P_c + \frac{(0.622)^2 L^2 e_c}{R T_c^2}} \right) \tag{40}$$

All subscripts C refer to the condensation level.

Condensation parameters of temperature, T_c, and pressure, P_c, are readily calculable from equation (41) and (42) after Edson [8]:

$$T_C = T_A - \frac{\gamma_d}{\gamma_d - \gamma_D} (T_A - T_D) \tag{41}$$

$$P_C = P_A \left(\frac{T_c}{T_A} \right)^{\frac{c_p}{R}} \tag{42}$$

For equation (40), the vapor pressure at the condensation level, e_c, must be calculated. Equation (43), the integrated form of the Clausius - Clapeyron equation may be used since air at the condensation level is saturated, and P_c and T_c are known from (41) and (42).

$$e_c = (6.107) \exp \left[\frac{5418.0}{273.16} - \frac{5418.0}{T_c} \right] \tag{43}$$

In like manner the vapor pressure of the sea surface, e_w, (from equation 32) is calculated since the sea surface is saturated (equation 44). Also, a correction factor of 0.98 is required for salinity effects.

$$e_w = (0.98)(6.107) \exp \left[\frac{5418.0}{273.16} - \frac{5418.0}{T_w} \right] \tag{44}$$

Finally, the advecting wind at 10-meters, $\mathbb{V}_{10}$, must be derived for use in equations (31) and (32). The geostrophic wind at the surface is taken as equal to the 1000 mb geostrophic wind. However, due to frictional interaction with the ocean surface, the geostrophic wind must be reduced in magnitude and rotated to the left, to yield the advecting frictional wind, $\mathbb{V}_{10}$. For the convective case the observed cross-isobar angle at mid-latitudes is approximately 15° [10].

Reduction of the 10-meter wind speed as expressed by the ratio, V/V_{g0}, where V_{g0} is the 10-meter geostrophic wind speed, may be obtained from a study by Lettau [13]. By assuming a roughness length of $z_0 = 0.1$ cm for oceanic areas, the 10-meter geostrophic wind ratio for neutral conditions is, $V/V_{g0} = 0.60$. Lettau then allows for stability criteria, and for the convective case a value of $V/V_{g0} = 0.7$ is arrived at. Similarly, $V/V_{g0} = 0.5$ for the inversion case.

Advection by the frictional wind is derived by first decomposing the vector advection as shown by equation (45):

$$\mathbb{V} \cdot \nabla T_A = u \frac{\partial T_A}{\partial x} + v \frac{\partial T_A}{\partial y} \tag{45}$$

Then multiplying by 0.7, and rotating the field through the angle $\alpha = 15^\circ$ by a rotational change of coordinates leaves equation (46):

$$\mathbb{V}_{10} \cdot \nabla T_A = 0.7 \left(u_{g0} \frac{\partial T_A}{\partial x} \cos\alpha - v_{g0} \frac{\partial T_A}{\partial y} \sin\alpha - v_{g0} \frac{\partial T_A}{\partial y} \cos\alpha + u_{g0} \frac{\partial T_A}{\partial x} \sin\alpha \right) \tag{46}$$

Assuming the geostrophic conditions, $u_{g0} = -\frac{g}{f}\frac{\partial z_0}{\partial y}$, $v_{g0} = \frac{g}{f}\frac{\partial z_0}{\partial x}$, equation (46) may be written in its final form as given by equation (47).

$$\mathbb{V}_{10} \cdot \nabla T_A = 0.7 \frac{g}{f} \cos 15^\circ \, \mathbb{J}(z_0, T_A)$$

$$- 0.7 \frac{g}{f} \sin 15^\circ \left(\frac{\partial z_0}{\partial x}\frac{\partial T_A}{\partial y} + \frac{\partial z_0}{\partial y}\frac{\partial T_A}{\partial x} \right) \quad (47)$$

Similarly, equation (48) is derived for the advection of vapor pressure by the 10-meter wind.

$$\mathbb{V}_{10} \cdot \nabla e_A = 0.7 \frac{g}{f} \cos 15^\circ \, \mathbb{J}(z_0, e_A)$$

$$- 0.7 \frac{g}{f} \sin 15^\circ \left(\frac{\partial z_0}{\partial x}\frac{\partial e_A}{\partial y} + \frac{\partial z_0}{\partial y}\frac{\partial e_A}{\partial x} \right) \quad (48)$$

8. Numerical Procedures

All finite-difference operators used are standard centered-differences of the type,

$$\nabla^2 A = \frac{m^2}{d^2} \nabla^2 A \qquad J(A,B) = \frac{m^2}{4d^2} \mathbb{J}(A,B)$$

$$\Delta_R A = \frac{m}{2d} \Delta_R A = \frac{m}{2d} \left[(\Delta_x A)^2 + (\Delta_y A)^2 \right]^{\frac{1}{2}}$$

with five-point grids having a mesh distance, d = 381 km at 60° latitude. Here, m is the map factor for polar stereograph projections and d is the grid spacing.

The variables needed for input into the diagnostic omega equation were scaled as follows:

$m = \hat{m} . 2$ $\qquad$ $P = \hat{P} . 2^{11}$ mb

$\eta = \hat{\eta} . 2^{-9}$ sec^{-1} $\qquad$ $e = \hat{e} . 2^{9}$ mb

$\omega = \hat{\omega} . 2^{8}$ mb/sec $\qquad$ $\sigma_H = \hat{\sigma}_H . 2^{35}$ cm^2/sec^2

$\omega_{LO} = \hat{\omega}_{LO} . 2^{8}$ mb/sec $\qquad$ $Z_T = \hat{Z}_T . 2^{20}$ cm

$D = \hat{D} . 2^{17}$ cm $\qquad$ $\dot{Q} = \hat{\dot{Q}} . 2^{10}$ gm-cm^2/sec^3

$T = \hat{T} . 2^{9}$ °C $\qquad$ $\hat{f} = 1.45842 \times 10^{-4} \sin\phi$ sec^{-1}

The following physical constants were also needed in the computations of ω. All values of those constants have been expressed in cgs units.

$d = 38{,}1000{,}000$ cm

$g = 980.0$ cm/sec^2

$R = .0287 \times 10^7$ cm^2/sec^2 - °K

$R_V = 0.461 \times 10^7$ cm^2/sec^2 - °K

$C_P = 1.003 \times 10^7$ cm^2/sec^2 - °K

$L = 2500 \times 10^7$ cm^2/sec^2 - °K

$\gamma_d = 0.9771 \times 10^{-4}$ °C/cm

$\gamma_D = 0.1600 \times 10^{-4}$ °C/cm

$\rho_0 = 1.213 \times 10^{-3}$ gm/cm^3

$\rho_1 = 1.055 \times 10^{-3}$ gm/cm^3

$\rho_2 = 0.919 \times 10^{-3}$ gm/cm^3

The scaled Helmholtz equation, which follows from equation (20), expressed in units of, mb - cm^2/sec^3 , is shown on next page.

$$\nabla^2(\hat{\bar{\sigma}}_H\hat{\omega}_1) - 4.5873\frac{\hat{f}\hat{\eta}_1 d^2}{\hat{\bar{\sigma}}_H\hat{m}^2}(\hat{\bar{\sigma}}_H\hat{\omega}_1)\cdot 2^{-46} =$$

$$-30.54\hat{f}\hat{\eta}_1\frac{d^2}{\hat{m}^2}\hat{\omega}_{L0}\cdot 2^{-46} - \frac{g^2\hat{P}_1\hat{m}^2\nabla^2}{f\ln\frac{P_0}{P_2}4d^2}\mathbb{J}(\hat{D}_1, \hat{D}_2-\hat{D}_0)\cdot 2^4$$

$$+\frac{g\hat{P}_1}{4\ln\frac{P_0}{P_2}}\left[\mathbb{J}(\hat{D}_0, \hat{\eta}_0) - \mathbb{J}(\hat{D}_2, \hat{\eta}_2)\right]\cdot 2^{-24} - pR\nabla^2\frac{\hat{\dot{Q}}}{C_P}\cdot 2^{-43} \qquad (49)$$

All height values have been replaced by D-values, the deviation of the height field from a standard atmosphere height.

The scaled stability parameter is given by equation (50) and is in units of cm^2/sec^2.

$$\hat{\bar{\sigma}}_H = \frac{R}{\ln\frac{P_0}{P_2}}\left\{\frac{g}{C_P}\left[(\hat{D}_2-\hat{D}_0)\cdot 2^{19} + 290000\right] - (T_0-T_2)2^{9}\right\}2^{-35} \qquad (50)$$

For computation of the stability parameter D is re-scaled to $D = \hat{D} \cdot 2^{19}$ to prevent overflow when adding the standard height, $Z_{700}-Z_{1000}$ = 290,000 cm. Stability calculations are based on 1000-700 mb differences and throughout the derivation all gradients and differences at the P_1 and P_2 levels are assumed equal to those at 850 mb and 700 mb respectively.

To prevent computational instability during the numerical Helmholtz solution a minimum value of stability is required.

This value corresponds to a maximum lapse of $\frac{1}{8}\gamma_d$:

$$\left(\hat{T}_0-\hat{T}_2\right)2^9 \doteq \frac{2}{8}\frac{g}{C_p}\left[\left(\hat{D}_2-\hat{D}_0\right)\cdot 2^{19}+290000\right]$$

Equation (51) is the scaled lower boundary condition in units of mb/sec.

$$\left(\hat{\omega}_{LO}\right)_n = -\rho_n \frac{g^2\hat{m}^2}{\hat{f}\,4d^2}\left(10^{-3}\right)\mathbb{J}\left(\hat{D}_n,\hat{Z}_T\right)2^{31}$$

$$-\rho_n\frac{g^3\hat{m}^3}{\hat{f}^3 2d^3}\left(10^{-3}\right)C_D\,\Delta_R\hat{D}_n\nabla^2\hat{D}_n\cdot 2^{29} \qquad (51)$$

Here, the subscript n refers to the terrain-height dependent cases, 1, 2, and 3 explained in section (5). The factor 10^{-3} converts dynes/cm^2 to millibars.

Equations for the diabatic term are computed in terms of heating rates, $\dot{Q}/C_p$, which are assigned to grid points according to the existing grid condition: convective, inversion, or neutral.

Equation (52) shows the scaled heating rate for the convective case.

$$\frac{\hat{\dot{Q}}}{C_p} = \left[\left(\frac{\hat{P}_A-\hat{P}_C}{3600}\right)\frac{\Delta\hat{T}_A}{\Delta t}+\left(\frac{\hat{P}_C-\hat{P}_2}{3600}\right)\frac{\Delta\hat{T}_{WB}}{\Delta t}\right]\cdot 2^{10} \qquad (52)$$

The factor, 3600, converts hours to second and heating rate is now in units of mb - $^\circ$C/sec.

Equation (53) shows the scaled heating rate for the inversion case (a cooling process) also in mb - $^\circ$C/sec.

$$g\left(\frac{\hat{Q}_c}{C_p}\right) = \frac{g^2\hat{m}}{\hat{f}\,2d}\,\frac{(0.5)(0.03)(4.184\times10^7)}{K\,C_p}\Delta_x\hat{D}_0(\hat{T}_W-\hat{T}_A)2^{17} \quad (53)$$

The factor (4.184×10^7) converts gm-cal/sec to dyne-cm/sec; $K = 24 \times 3600$ which converts 24-hours to seconds; 0.03 replaces 3.0, and permits velocity computation to be made in cm/sec.

Land area grid points are masked and receive heating rate values of zero as do oceanic grid points which satisfy the neutral conditions. The remaining oceanic grid points receive either heating or cooling values depending on the prevailing condition at the point. The diabatic term is then computed and the scaled form in units of mb-cm^2/sec^3 is shown by equation (54).

$$-\frac{R}{\ln\frac{P_0}{P_2}}\nabla^2\frac{\hat{\dot{Q}}}{C_p}\cdot 2^{-33} \quad (54)$$

Here, P_0 approximates P_A for ease of computation.

Other equations involved in the computation of the diabatic term are scaled as follows:

(1) The air temperature change is units of $^\circ$C/hr.

$$\frac{\Delta\hat{T}_A}{\Delta t} = 0.12(\hat{T}_W-\hat{T}_A) - (0.10)2^{-9}$$

$$-0.7\frac{g\hat{m}^2}{\hat{f}4d^2}\left[\cos 15^\circ J(\hat{z}_0,\hat{T}_A) - \sin 15^\circ(\Delta_x\hat{z}_0\Delta_y\hat{T}_A + \Delta_y\hat{z}_0\Delta_x\hat{T}_A)\right]2^{19} \quad (55)$$

(2) The dew-point temperature change in units of °C/hr.

$$\frac{\Delta \hat{T}_D}{\Delta t} = \frac{\Delta \hat{T}_A}{\Delta t} + \frac{0.622L}{c_p \hat{P}_A}\Big\{0.15(\hat{e}_W - \hat{e}_A) - (0.18)2^{-9}$$

$$- 0.7\frac{g\,\hat{m}^2}{\hat{f}\,4d^2}\Big[\cos 15^\circ J(\hat{z}_0, \hat{e}_A) - \sin 15^\circ(\Delta_x \hat{z}_0 \Delta_y \hat{e}_A +$$

$$+ \Delta_y \hat{z}_0 \Delta_x \hat{e}_A)\Big]2^{19}\Big\}2^{11} - \frac{(0.622L^2\hat{e}_S)}{c_p \hat{P}_A R_v(\hat{T}_A \cdot 2^9 + 273.16)^2}\frac{\Delta \hat{T}_A}{\Delta t}2^{-2} \tag{56}$$

(3) The wet-bulb temperature change in units of °C/hr.

$$\frac{\Delta \hat{T}_{WB}}{\Delta t} = \frac{\Delta \hat{T}_D}{\Delta t} + \left(\frac{\gamma_m - 0.1600}{0.8171}\right)\left(\frac{\Delta \hat{T}_A}{\Delta t} - \frac{\Delta \hat{T}_D}{\Delta t}\right)$$

where,

$$\gamma_m = -0.9771\,\frac{\hat{P}_c \cdot 2^{11} + \dfrac{0.622L\hat{e}_c \cdot 2^9}{R(\hat{T}_c \cdot 2^9 + 273.16)}}{\hat{P}_c \cdot 2^{11} + \dfrac{0.622L^2\hat{e}_c \cdot 2^9}{C_p R_v(\hat{T}_c \cdot 2^9 + 273.16)^2}} \tag{57}$$

(4) The condensation level temperature in units of °C.

$$\hat{T}_C = \hat{T}_A - \left(\frac{0.9771}{0.8171}\right)(\hat{T}_A - \hat{T}_D) \tag{58}$$

(5) The dew-point temperature in units of °C.

$$\hat{T}_D = \hat{T}_A - \frac{0.622L}{C_p \hat{P}_A}(\hat{e}_S - \hat{e}_A) \tag{59}$$

(6) The condensation level pressure in units of millibars, $\hat{P}_c = (\hat{P}_{c1} + \hat{P}_{c2})/2$

and,

$$\hat{P}_{C1} = \hat{P}_A\left(\frac{\hat{T}_{c1} \cdot 2^9 + 273.16}{\hat{T}_A \cdot 2^9 + 273.16}\right)^{\frac{C_p}{R}} \tag{60}$$

Here $\hat{T}_{c1}$ is computed using values of $\hat{T}_A$ and $\hat{T}_D$ in equation 58.

$$\hat{P}_{C2} = \hat{P}_A\left[\frac{T_{c2} \cdot 2^9 + 273.16}{\left(\hat{T}_A + \frac{\Delta \hat{T}_A}{\Delta t}\right)2^9 + 273.16}\right]$$

Here $\hat{T}_{C2}$ is calculated using values of $\hat{T}_{A1} + \Delta \hat{T}_A / \Delta t$ and $\hat{T}_{D1} + \Delta \hat{T}_D / \Delta t$ in equations (58), (59), (55), and (56).

9. The Computer Program

The Control Data Corporation 1604 digital computer was used in this study. It has a core capacity of 32,768 words of 48 bits each. An operational field of 1,977 grid points forming a 51 X 47 octagon inscribed within the 9°N latitude circle was employed. The grid-mesh is 381 km true at 60°N latitude.

Boundary conditions around the octagonal grid were determined by the standard subroutines used. Laplacian, Jacobian, and all other five-point center-difference operations set the grid boundary to zero. The Helmholtz relaxation operation set the edge and the next interior border point to zero.

Equation (49) was solved by a two-dimensional Liebmann relaxation technique wherein the $(n+1)$-iterate for any point is given by.

$$A_{ij}^{n+1} = A_{ij}^{n} + \frac{\lambda}{4}\left[\frac{\nabla^2 A_{ij} - (AB)_{ij} - C_{ij}}{z^{-2} B_{ij} + 1}\right]^n$$

Here, λ is the over-relaxation coefficient and the residual at any step, R_n, is,

$$R_n = \frac{\lambda}{4}\left[\frac{\nabla^2 A_{ij} - (AB)_{ij} - C_{ij}}{z^{-2} B_{ij} + 1}\right]^n$$

The over-relaxation coefficient used was, $\lambda = 1.414$, which allowed convergence in approximately 30 scans over the 1977 point grid using an initial guess field of zero. The convergence criterion is that the iteration ceases when $\epsilon^{(n)}$ defined by $\epsilon^{(n)} \equiv A^{(n+1)} - A^{(n)}$ falls below 3000 mb-cm^2/sec^3. This value corresponds to a vertical velocity of 0.3×10^{-4} mb/sec for a stability of 83×10^6 cm^2/sec^2 (the standard atmosphere stability).

Total computation time was approximately 1 minute and 30 seconds with 23 seconds required for the heating term and 26 seconds for the boundary condition.

Due to the abrupt cut-off criteria used for delineating the three cases of diabatic heating (inversion, convective, neutral) the final $\dot{Q}/c_p$ field was smoothed with a five-point smoother of the form,

$$\bar{A} = A + K\nabla^2 A$$

The smoothing coefficient k is a constant value of (1/8) and the field was smoothed twice. The smoothing operation removed small scale irregularities caused by the cut-off criteria of the heating term but also reduced peak values.

A standard FNWF filtering process followed all operations involving the Laplacian. This process removes all small scale features with wave numbers greater than 15 at latitude 45°.

10. Results and Conclusions

A series of four successive 12-hourly data-sets beginning with the 00Z maps of 28 April 1966 and ending with 12Z, 29 April 1966 were used. Since the only diabatic heating mechanism introduced into this study was that arising by "conduction" from the underlying oceanic surface, the results have been depicted only over the North Atlantic Ocean. This region has a large density of reporting ships and thus the reason for its selection.

The computational omegas and their associated parameters for 00Z 28 April 1966 are contained in figures (6) through (11) which are appended. Of these, not only the diabatic omegas are shown, but also, the additive effect of the diabatic influence as contrasted with adiabatic computations. In figures (12) through (17) only the final-product omegas of this study are shown together with the corresponding FNWF analyzed 850 mb D-fields to serve as identifiers, that is, to indicate qualitative coherency between the vertical motions and the associated motion systems.

In the sequence of figures (6) through (11), it is of interest to note the pattern similarity between comparative adiabatic vertical motion computations, those produced here, and, for the same times, those produced by FNWF, which are based upon the procedure described by Haltiner et al [9]. The magnitude and position of the updraft-centers show a strong similarity with those of FNWF, however, the model described by this paper shows larger downdraft areas extending east-

ward from Newfoundland. This apparent discrepancy may be attributed to the two differing treatments in ω_{Lo}(figures 18 and 19). The ω_{Lo} at the continent-edge essentially becomes the boundary condition for the oceanic computations.

On the other hand, however, one of the major objectives of this study was to test a simplified ω_{Lo} which uses only parameters pertaining to the standard levels of analysis (equation 23). In this connection, the greatest time-consuming aspect of the FNWF ω-computation is that of deriving ω_{Lo}, and associated parameters at terrain height requiring pressure extrapolation. The general similarity in the fields of ω_{Lo} by the two operational procedures which have been discussed is evident by an inspection of figures (18) and (19), and this can perhaps be a justification for continuing the simpler ω_{Lo} computations.

The diabatic effects, which were obtained for the 00Z April 28 synoptic time, depict values of $\dot{Q}/Cp$, so that the large positive center southeast of Newfoundland is realistic with regard to the northerly flow passing over the Gulf Stream. The maximum heating value for the four map periods over the Gulf Stream is $\dot{Q}/Cp = 0.067$ mb - °C/sec, which corresponds to a layer mean heating rate of 0.3 °C/hr for the 1000-700 mb layer. The equivalent thickness change for this layer is approximately 3 meters/hr.

For comparison, Petterssen's values [17] for the 1000-500 mb thickness change during a similar synoptic situation in late March show a maximum value of 6 to 8

meters/hr in the vicinity of the Gulf Stream. These values for sensible and latent heating were computed using flux calculations at the surface based on empiricisms similar to those of Laevastu. It should be noted that heating rates for this time period in late March are indeed greater than late April values.

Finally, with regard to the diabatic term, note that in the area of qualitative verification no significant diabatic cooling values were observed. These areas are limited in extent in winter and spring, but could be of greater synoptic consequence in the summer and fall seasons.

The effect of the diabatic term as it appears on the forcing function side of equation (24) gives rise to figure (11), applicable at approximately 850 mb, but actually representing the layer-mean $\bar{\omega}$. As could be anticipated from the standpoint of heat injection into moving parcels, the change (inclusion of the diabatic term) has been such as to expand the southern rim of the trough which extends towards the southwest from Iceland. Similar aspects of coherency between the <u>resultant</u> ω of this study and the trough movement and development may be traced out, feature by feature as one proceeds through the synoptic sequences. For example, the pronounced trough over the Atlantic has, by 00Z of 29 April, become oriented North-South just east of Greenland. At the same time the updraft cell has taken on this same orientation just west of the same trough, so that the $\bar{\omega}$-field gives some confirmation of the dynamic

processes involved in these map changes.

The model for vertical motion, as presented by this paper, appears to be quite sucessful and future plans involve inclusion of a more realistic frictional term. These more realistically computed omegas (extended to 500 mb) are then to be used for feedback to yield a prognostic z-field, hour-by-hour. The ultimate aim, of course, is to realize a smaller R.M.S. error as the motion systems progress over the ocean.

11. Acknowledgements

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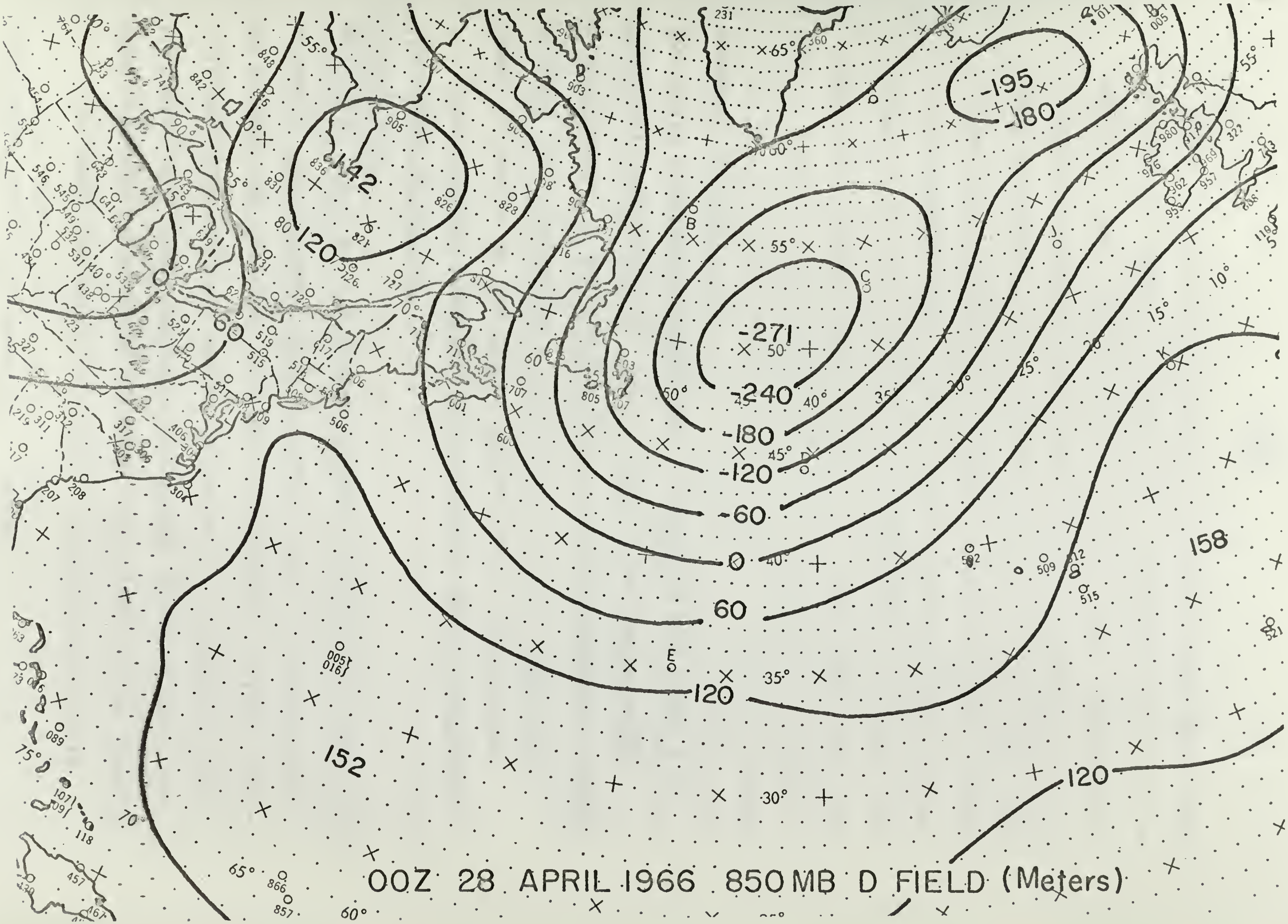
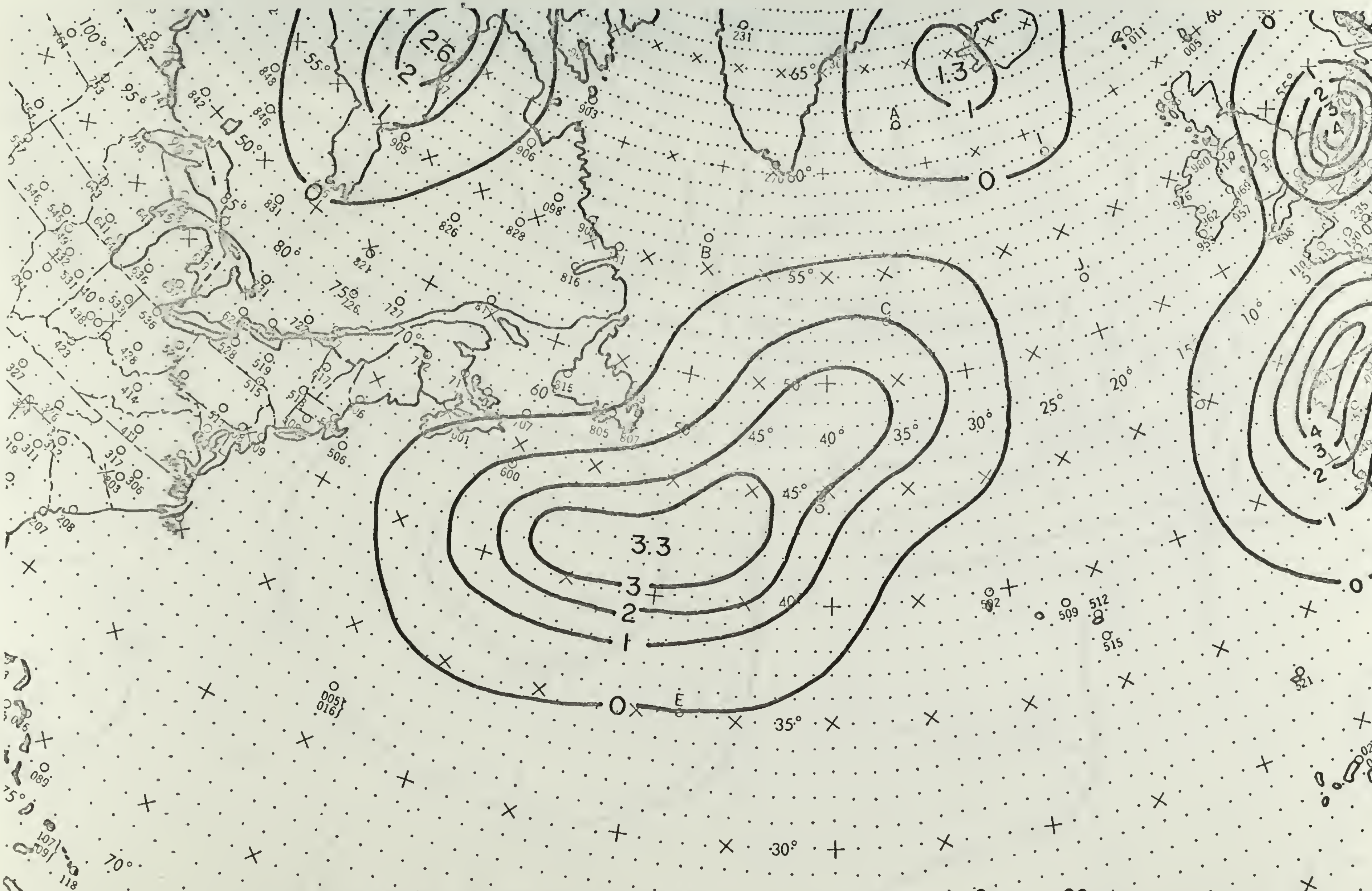


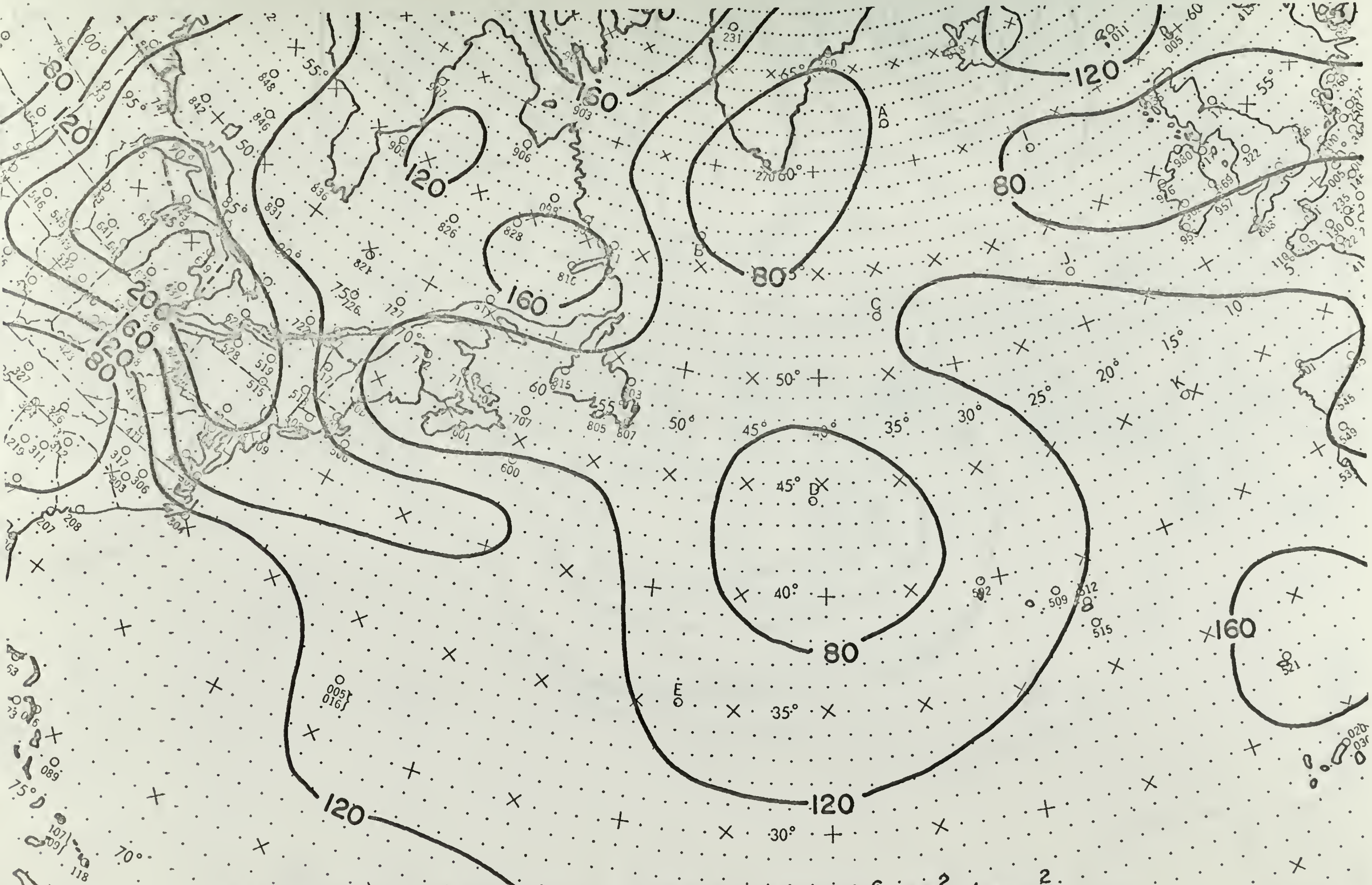
OOZ 28 APRIL 1966 850 MB D FIELD (Meters)

Figure 6



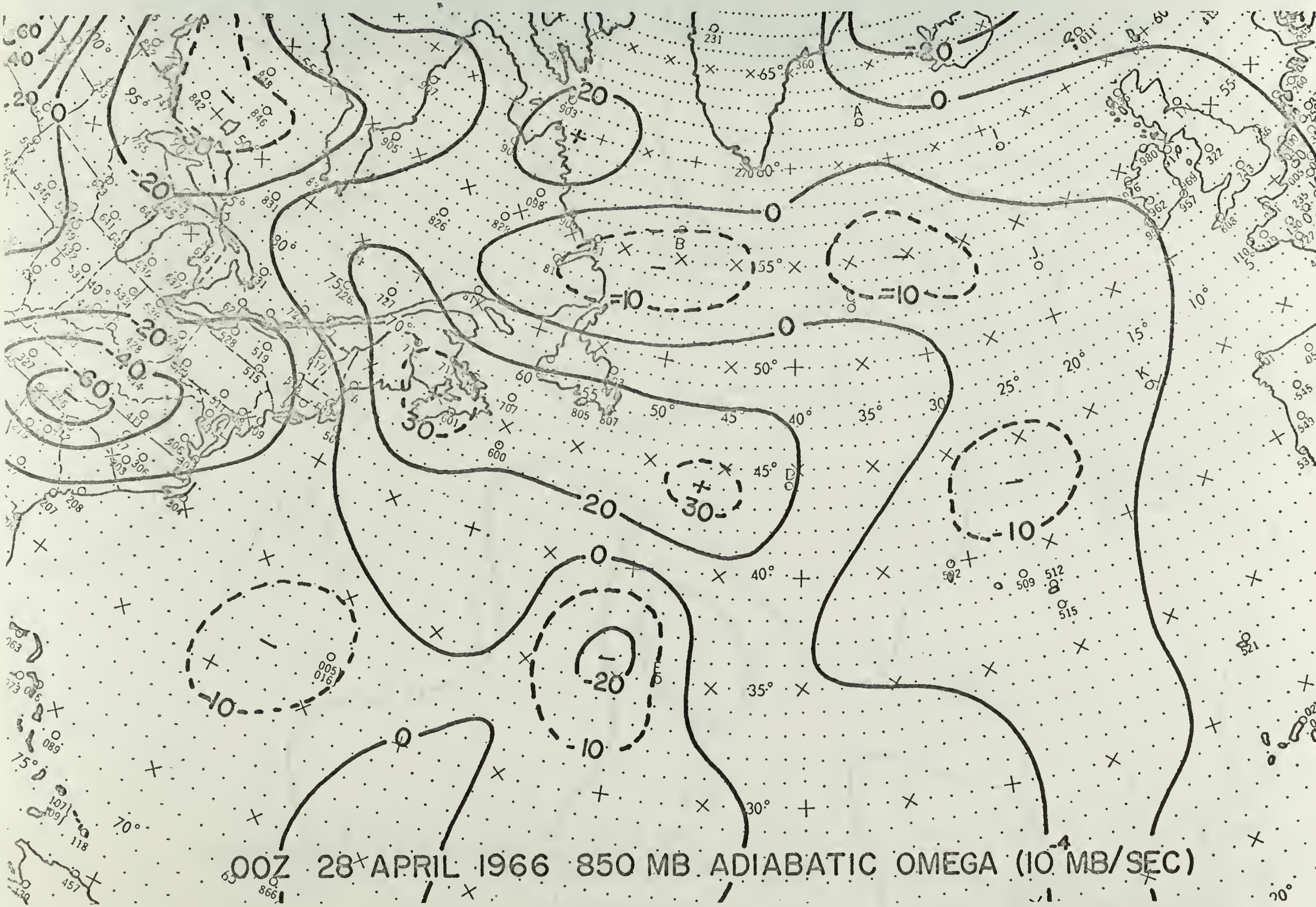
00Z 28 APRIL 1966 DIABATIC HEATING (10^{-2} °C MB-O/SEC)

Figure 7



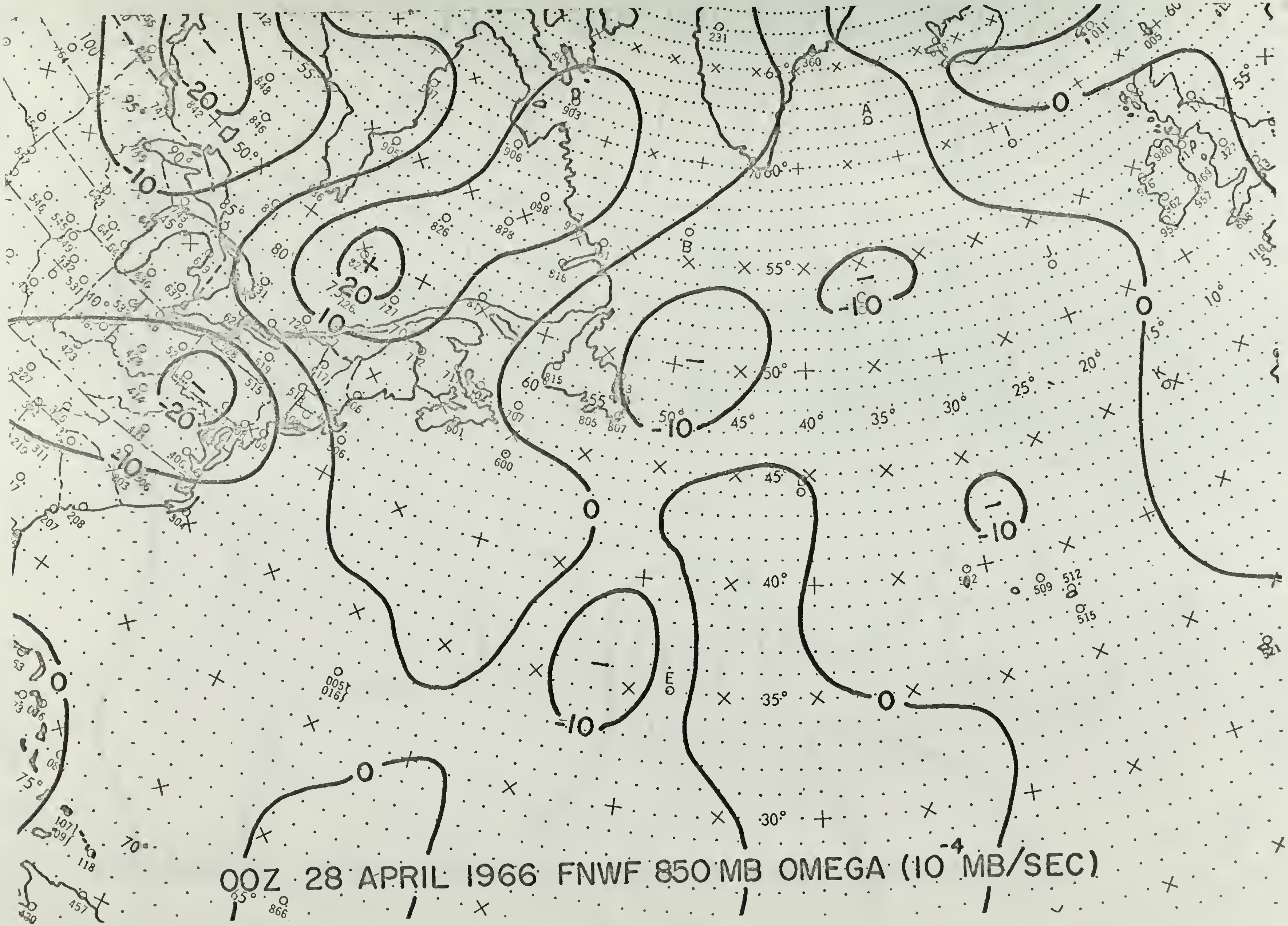
00Z 28 APRIL 1966 STABILITY (10^6 CM^2/SEC^2)

Figure 8



00Z 28 APRIL 1966 850 MB ADIABATIC OMEGA (10^{-4} MB/SEC)

Figure 9



00Z 28 APRIL 1966 FNWF 850 MB OMEGA (10^{-4} MB/SEC)

Figure 10

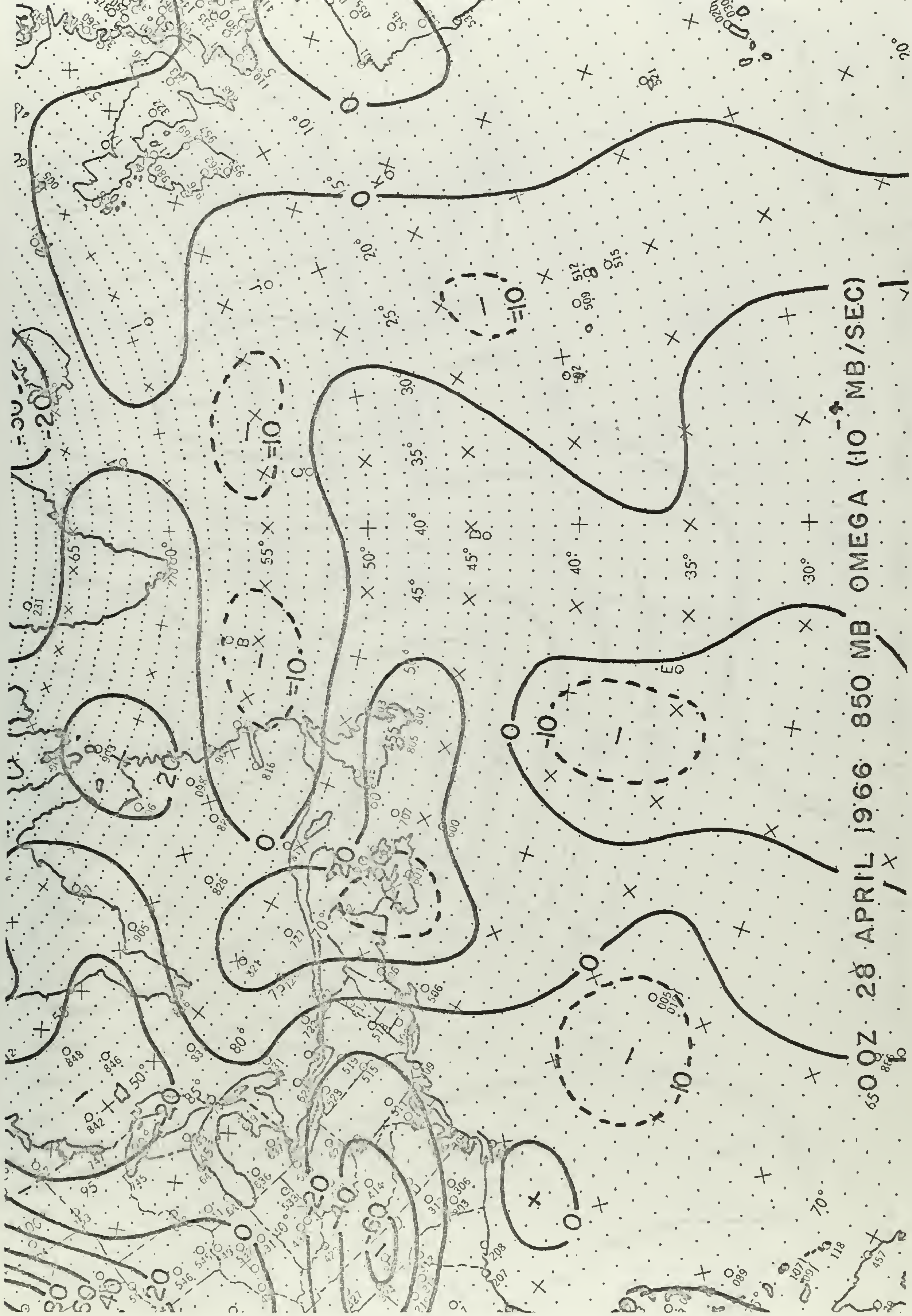
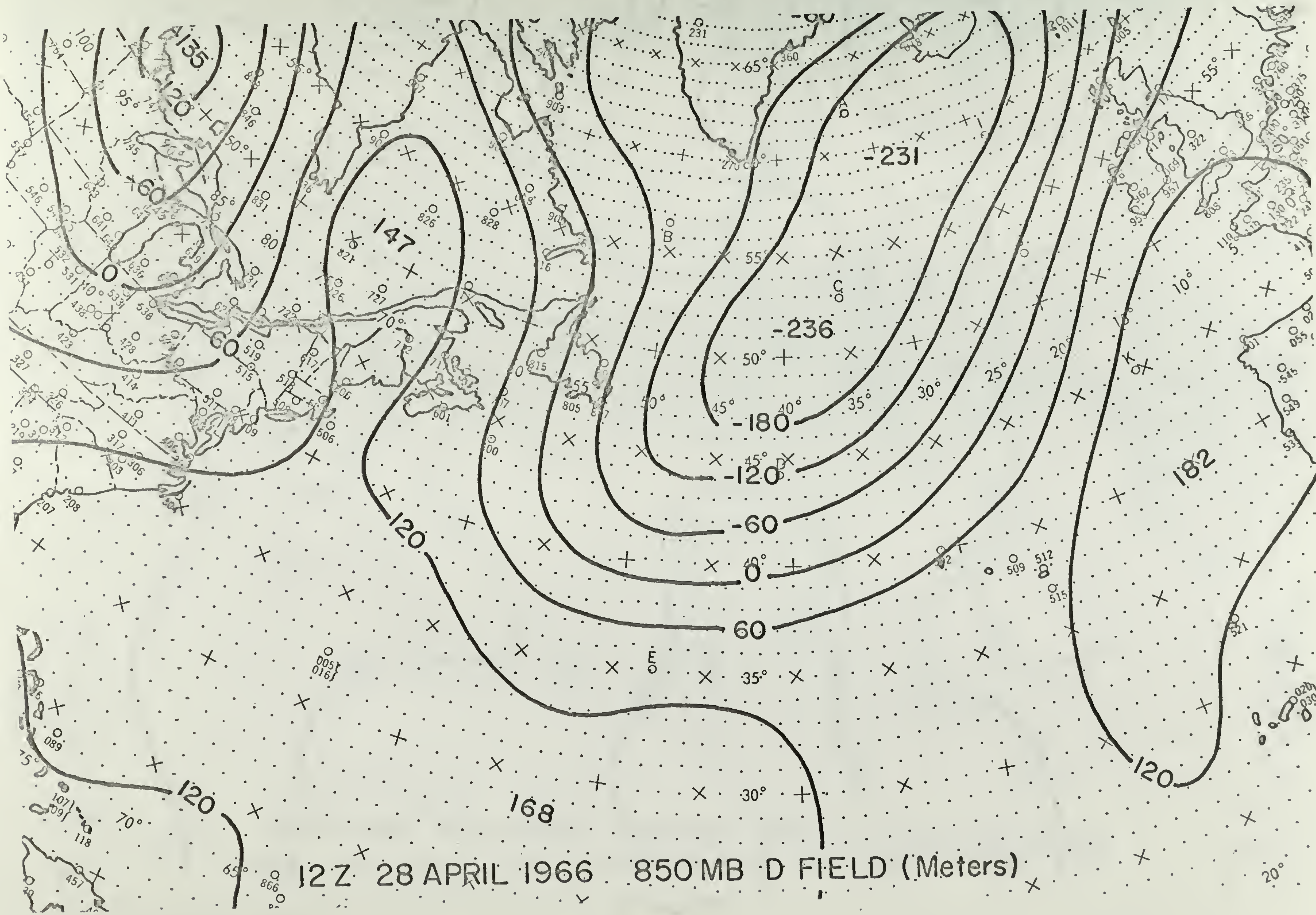


Figure 11



12Z 28 APRIL 1966 850MB D FIELD (Meters)

Figure 12

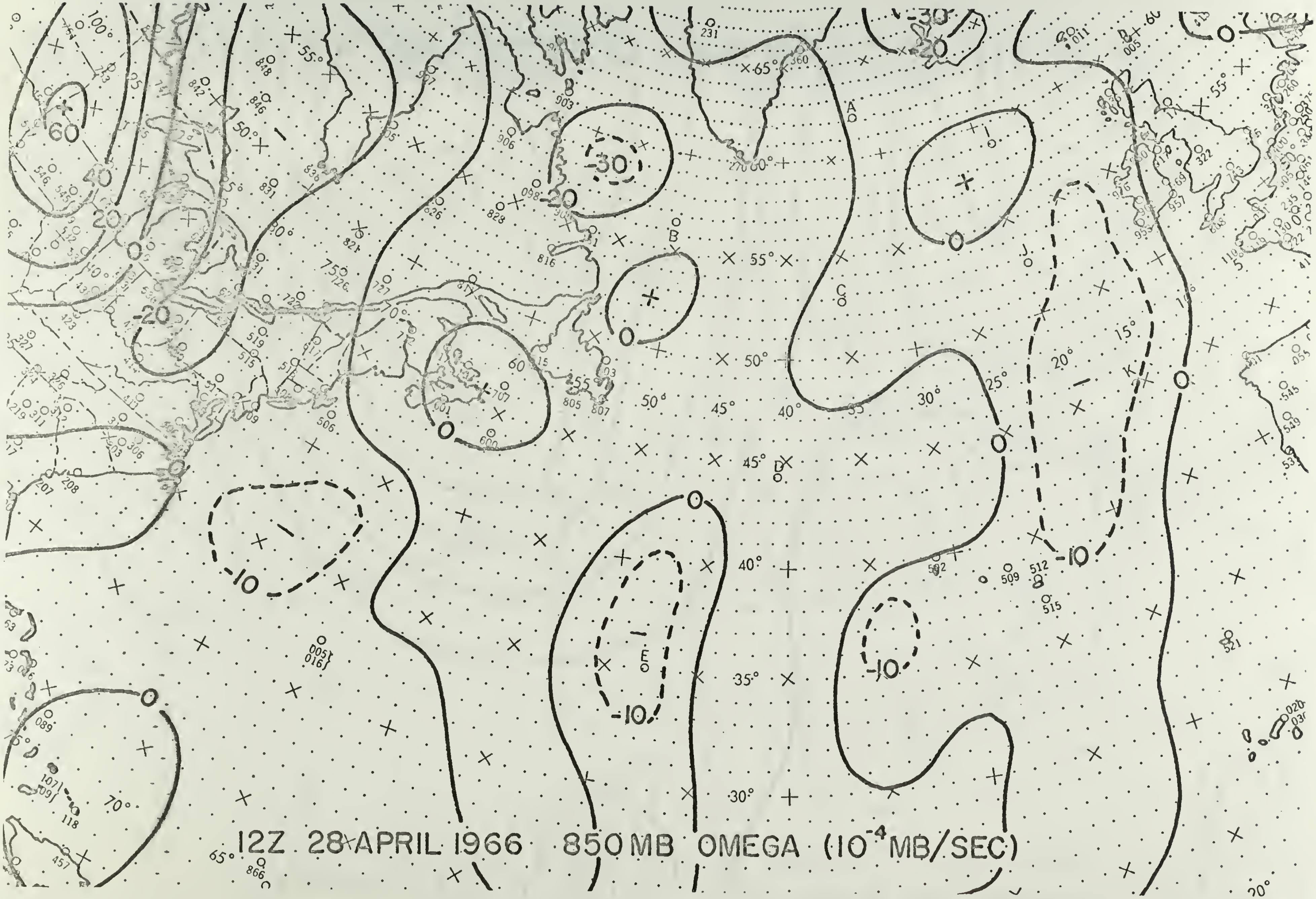


Figure 13

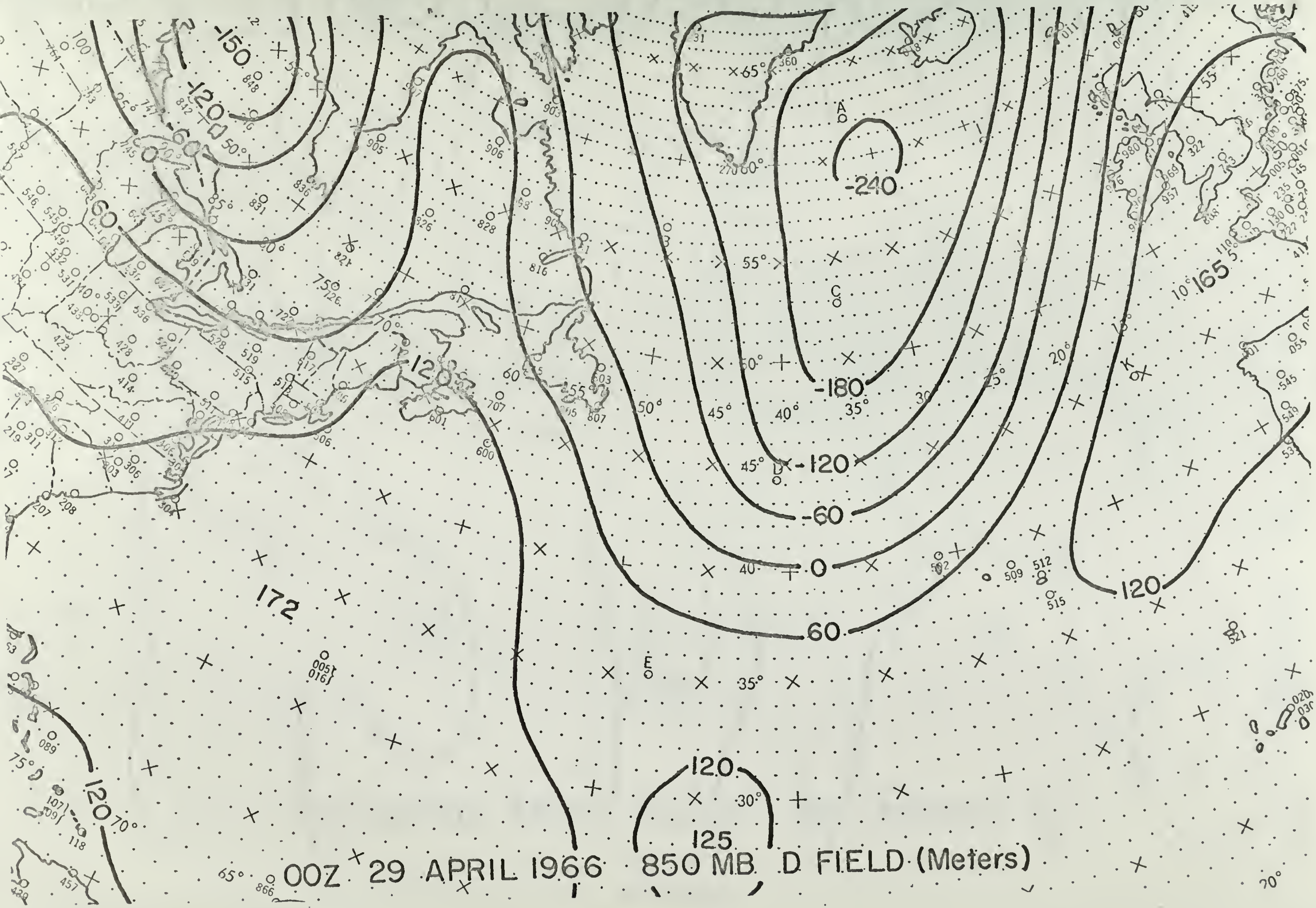
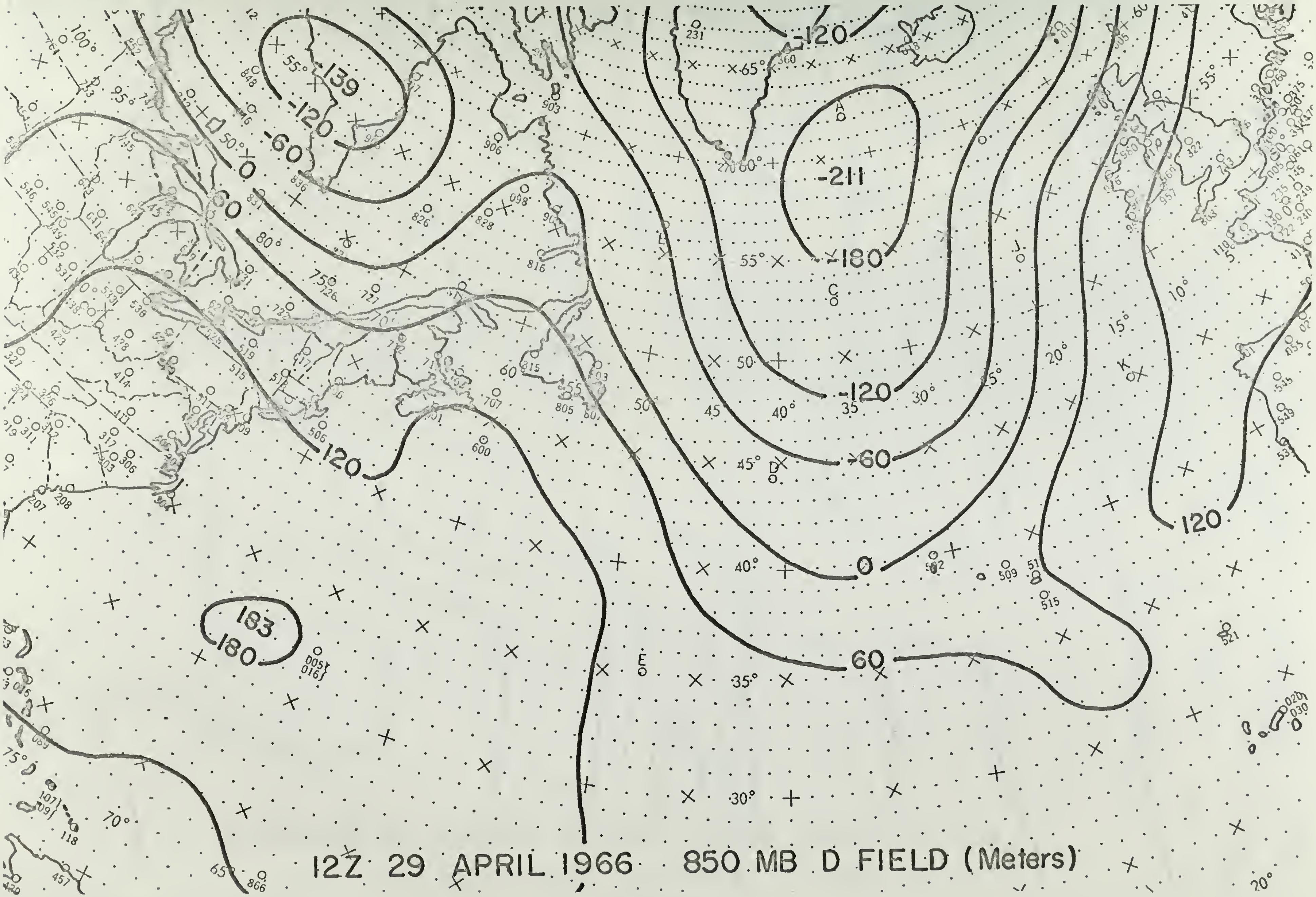

-150
-120
-60
60
120
172
-240
-180
-120
-60
0
60
120
165
120
125
00Z 29 APRIL 1966 850 MB D FIELD (Meters)

Figure 14

Figure 15



12Z 29 APRIL 1966 850 MB D FIELD (Meters)

Figure 16

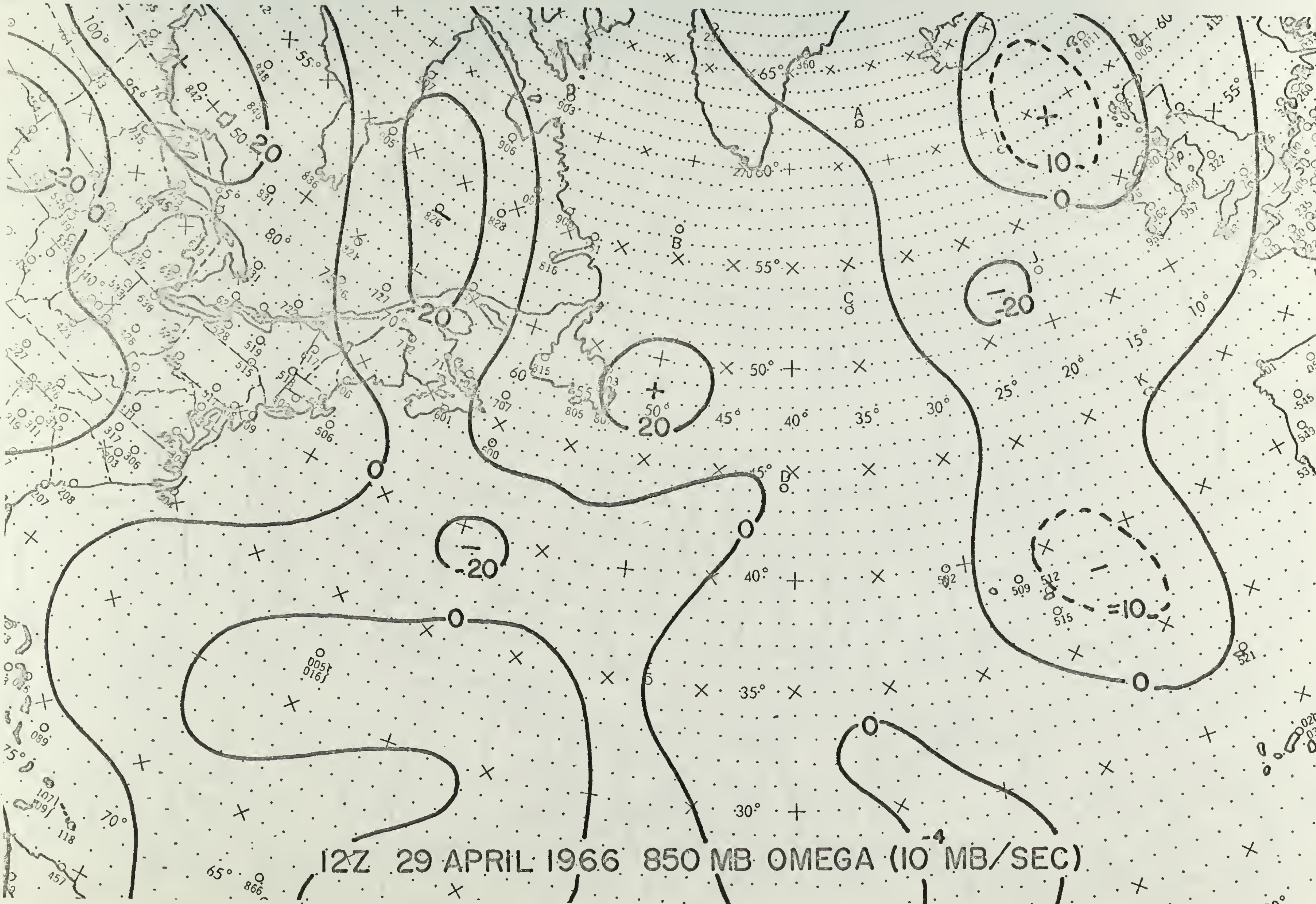


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Figure 17

Figure 18

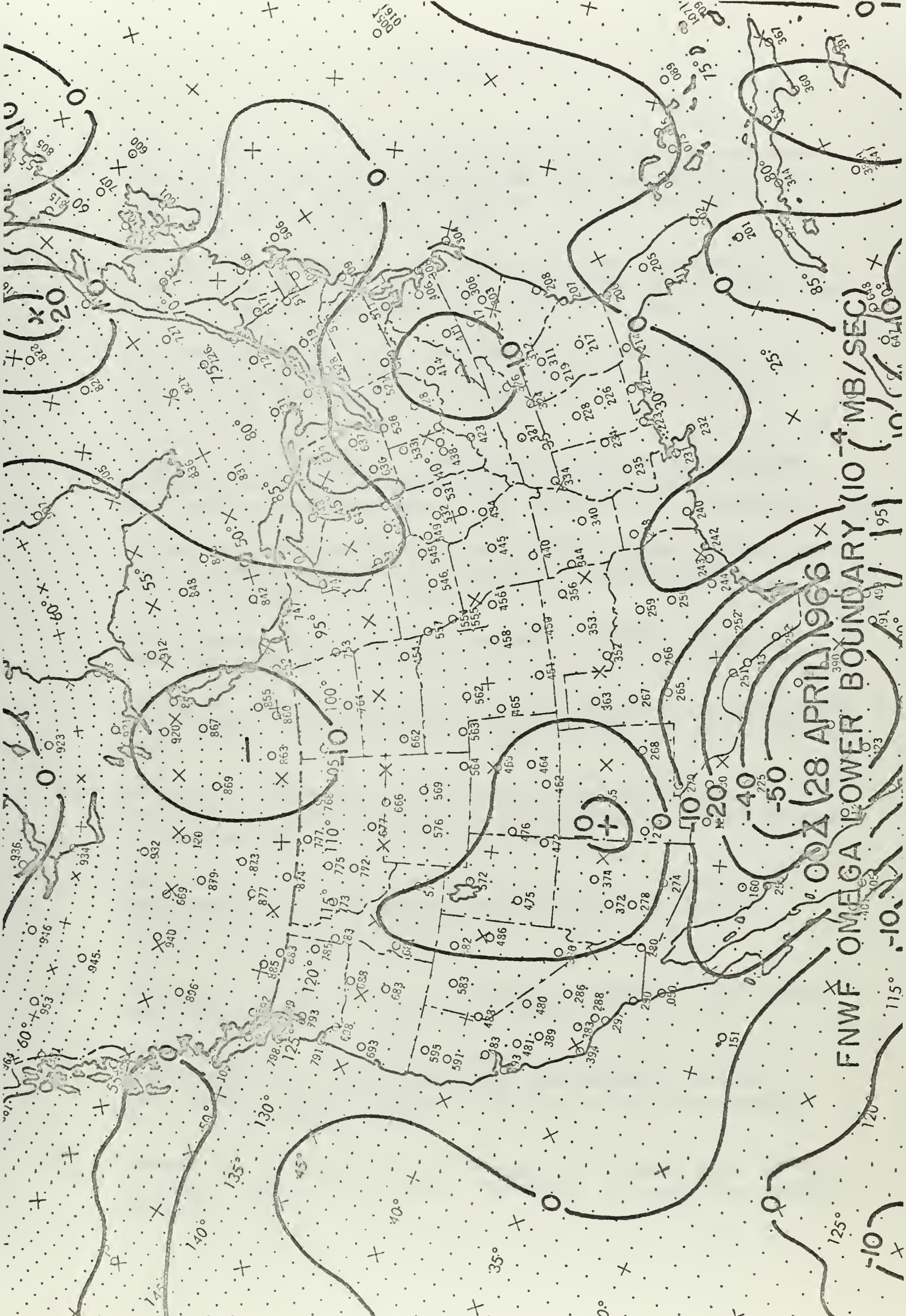

Figure 19

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MASTER OF SCIENCE THESIS (METEOROLOGY)

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A two-dimensional omega equation is derived by combination of the vorticity and thermodynamic equations. The desired omega is then taken to be the logarithmic average in the 1000-700 mb layer. A diabatic term, after Laevastu, for oceanic areas only is included to deduce the empirical temperature and vapor-pressure changes associated with sensible and latent heating in the maritime layers. Over both continental and oceanic areas a frictional vorticity sink is included in order that excessive energy cannot be generated over the ocean. Among other novel features is the use of the Holl static-stability parameter which affords vertical consistency in the analyses prepared by Fleet Numerical Weather Facility.

14. KEY WORDS	LINK A		LINK B		LINK C	
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OMEGA EQUATION						
VERTICAL MOTION						
DIABATIC HEATING						

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